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Mixed Taper Equations for African Mahogany Plantations (*Khaya grandifoliola* C. DC.) Near Thinning and Clear-cut Ages

Ximena Mendes de Oliveira[1]
Andressa Ribeiro[2]
Rafaella Carvalho Mayrinck[3]
Antonio Carlos Ferraz Filho[2]

[1]Universidade Federal Rural da Amazônia, Campus Parauapebas, Engenharia Florestal, Parauapebas, PA, Brasil
[2]Universidade Federal do Piauí, Campus Professora Cinobelina Elvas, Engenharia Florestal, Bom Jesus, PI, Brasil.
[3]University of New Brunswick, Forestry, Fredericton, New Brunswick, Canada.

Abstract

The present study aimed to develop taper equations for African mahogany trees in two important ages (near to thinning and near to clear-cut ages), using, respectively, 100 seven-year-old trees were scaled in Minas Gerais and 46 fourteen- and fifteen-year-old trees were scaled in Pará. The fifth-degree polynomial, the polynomials with integer and fractional powers, the Kozak exponential model, and the modified Kozak model were tested. The equations were assessed and, next, the inclusion of a random term in the best equation at tree level was tested. The best resulting equation was selected, and it was validated using the bootstrapping method. The Kozak exponential model and the modified Kozak model were selected, to trees near thinning and near clear-cut age, respectively. Inserting a random term at tree level in the equation improved its estimates by 5.2% and 8.9% for trees near thinning and clear-cut ages, respectively.

Keywords: mixed models, random term, taper function, silvicultural ages.

1. INTRODUCTION

The genus *Khaya* belongs to the Meliaceae family, native to the African continent, and its species are known as African mahogany. These species drive great economic interest because of its wood quality and beauty (ITTO, 2023). African mahogany wood is used to produce high-end furniture, music instruments, in civil construction and naval industry (Pinheiro et al., 2011; Reis et al., 2019).

The first African mahogany trees were planted in Brazil in the Northern region, in 1976. Since then, the growing demand for tropical wood is leading to an increase in plantations around the country (Ribeiro; Ferraz Filho; Scolforo, 2017). This rise in African mahogany plantations lead to the need to build knowledge about the development of African mahogany plantations around the country, in several circumstances, under several management regimes, and taking into account the effects of different management regimes in the stem shape.

There are in the literature some studies about *Khaya grandifoliola*, initially wrongly identified as *Khaya ivorensis*, as clarified by Ferraz Filho et al. (2021). For example, about diameter distribution (Mayrinck et al. 2018), tree height modeling (Ribeiro et al. 2018a), volume modeling (Oliveira et al. 2018), site index classification (Ribeiro et al. 2016) and economic and risk analysis (Ribeiro et al., 2018b). However, there is little information about taper modeling and assortments for African mahogany trees, for example Costa et al. (2023), which does not cover different regions in the country, as we propose in this study.

Taper is affected by different stand attributes, such as species, age, spacings, and site index (Burkhart; Tomé, 2012). Modeling taper is an important tool for forest management, allowing to predict assortments (Nicoletti et al., 2020), which is important for wood trade and planning over the years. Different modeling approaches and fitting techniques are frequently tested, aiming to increase accuracy of models. Taper modeling can be stratified into three kinds of categories: simple models, models with variable exponent, and segmented models (Koirala et al., 2021). In Brazil, great part of the studies uses simple models (e.g., Figueiredo Filho et al., 2015;

Kohler et al., 2016; Teo; Esteves, 2022), being the most common the fifth-degree polynomial (Schöepfer, 1966). However, Liu et al. (2020a) refers to the variable exponent model of Kozak (Kozak, 1988) as the most studied globally. The segmented model also receives great attention because of its ability to describe specific portions of the stem along the tree (Terra et al., 2017), used when data is composed of bottom-to-top diameter measurements as in indirect scaling.

To better model taper and precisely determine assortments, there are different models and modeling approaches to be tested. Besides, there is the option of including different random effects in the models, resulting in a mixed effect model, which improves fitting in most cases. For example, models were improved for *Pinus taeda* trees in the South of Brazil (Santos et al., 2021a), for *Tectona grandis* in Center-West (Lanssanova et al. 2020), and for *Eucalyptus* all over Brazil (Scolforo et al. 2018).

Due to the importance of African mahogany plantations in Brazil and given the research gaps mentioned above, we hypothesize that: 1) variable exponential models may be superior to simple exponential models; and 2) using tree profile equations with random effects at the tree level will be a successful strategy to remove the heteroscedasticity and autocorrelation problems from the data structure used to build the tapering models. By testing these two hypotheses, we aim to achieve the goal of developing equations for two important ages of a plantation: near thinning and near clearcut. The equations were fitted following the steps: a) fit four models (two simple exponential and two variable models) and select the best one; b) include a random term in the selected model. The analysis was done considering two datasets, ages near thinning (7-year-old plantations in Minas Gerais) and near clearcut (14- and 15-year-old plantations in Pará).

2. MATERIAL AND METHODS

2.1. Data characterization

Data was collected from African mahogany plantations (*Khaya grandifoliola*) located in Piumhi municipality, Minas Gerais state (MG) (20.42º S and 46.02 W) and Santo Antônio do Tauá municipality, Pará state (PA) (1.18º S and 48.13º W). The climate in Piumhi is subtropical with dry summers (Cwa), with average minimum and maximum temperatures around 18ºC and 22ºC, respectively, and precipitation around 40 mm in the driest month. Santo Antônio do Tauá presents tropical climate (Af), with average temperature higher than 18ºC in the coldest month and no dry season, with precipitation higher than 60 mm in the driest month (Alvares et al., 2013).

The information about planting year, planting spacing, other associated species and age scaling were described in Table 1. According to Ferraz Filho and Ribeiro (2019), clear-cut ages for African mahogany plantations should take place at least 20 years after planting, and thinning should be defined based on forest growth stagnation, identified by the forest inventory. However, due to the low tree density applied in PA, the trees grew to sizes which could be submitted to clear cut (Figure 1), even at its current younger age. Therefore, the database was stratified into a) first thinning: 7-year-old trees in MG; c) clear cut: 14 and 15-year-old trees in PA.

Table 1. Planting year, planting spacing, other associated species and age scaling information about the data located in Piumhi municipality, Minas Gerais state (MG) and Santo Antônio do Tauá municipality, Pará state (PA).

Local	Planting year	Planting spacing	Other associated species	Age scaling
MG	2009	5,5 x 6 m	coffee plants	7 years
PA	2000	12,0 x 12,0 m	black pepper	14 – 15 years

Indirect scaling was performed using a Criterion RD 1000 with a TruPulse 200 hypsometer. The accuracy of indirect scaling is proved in the literature for eucalyptus (Curto et al., 2019), native species (Stenman et al., 2023) and African mahogany (Oliveira et al., 2018). The scaled trees belonged to different diameter (dbh) and height (h) classes were defined based on forest inventories data to better represent the population (Mugasha et al. 2016). A total of 100 trees were scaled in MG and 46 trees were scaled in PA (Figure 1).

To control the bias of indirect scaling, diameters up to 2,0 m were simultaneously measured directly and indirectly to calibrate the operator, as in Santos et al. (2021a). The diameters over the stem were measured at 0.1; 0.7; 1.3 and 2.0 m and every 1 meter after that up to the stem height (h_f), at the first bifurcation in the stem.

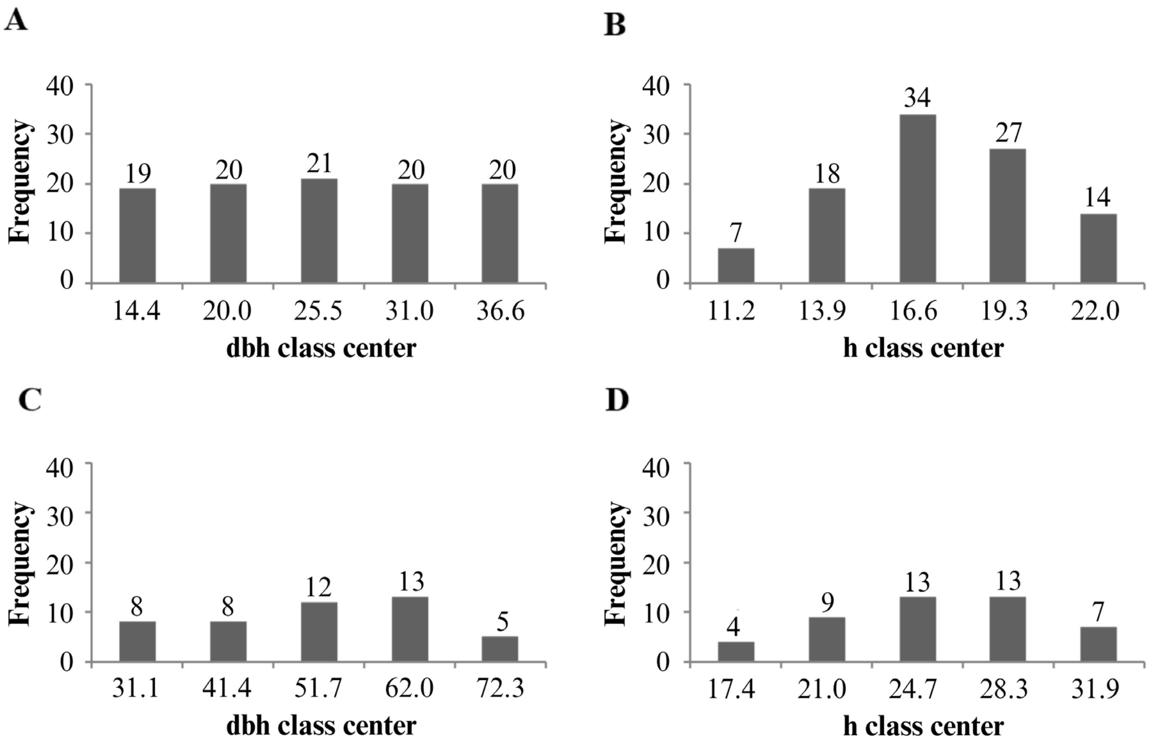

Figure 1. Frequency of trees near thinning scaled according to diameter class (A) and tree height (B) and frequency of trees near clear-cut scaled according to diameter class (C) and tree height (D).

2.2. Models

Four models were tested: (1) fifth degree polynomial (Schöepfer, 1966); (2) integer and fractional powers (Hradetzky, 1976); (3) Kozak exponential model (Kozak, 1988) and (4) Kozak modified exponential model (Kozak, 2004). The models (1) and (2) are normally fitted with the intercept (e.g., Figueiredo Filho et al., 2015; Kohler et al., 2016). However, in this study, the intercept was not considered. The constrain when (Scolforo et al., 2018; Santos et al., 2021a) was applied. The powers ($p_{i's}$) tested for model (2) were the 55 considered by Assis et al. (2002), varying from 0,00001 to 95. This model was fit using the Stepwise method, and best equation was selected.

$$\frac{d_i}{dbh} = \beta_1\left(1-\frac{h_i}{h}\right) + \beta_2\left(1-\frac{h_i}{h}\right)^2 + \beta_3\left(1-\frac{h_i}{h}\right)^3 + \beta_4\left(1-\frac{h_i}{h}\right)^4 + \beta_5\left(1-\frac{h_i}{h}\right)^5 + \varepsilon_i \quad (1)$$

$$\frac{d_i}{dbh} = \beta_1\left(1-\frac{h_i}{h}\right)^{p_1} + \beta_2\left(1-\frac{h_i}{h}\right)^{p_2} + \cdots + \beta_n\left(1-\frac{h_i}{h}\right)^{p_n} + \varepsilon_i \quad (2)$$

$$d_i = a_0 dbh^{a_1} a_2{}^{dbh} x_{1988}{}^{\left[\beta_1 z^2 + \beta_2 \ln(z+0{,}001) + \beta_3\sqrt{z} + \beta_4 \exp(z) + \beta_5\left(\frac{dbh}{h}\right)\right]} + \varepsilon_i \quad (3)$$

$$d_i = a_0 dbh^{a_1} h^{a_2} x_{2004}{}^{\left[\beta_1 z^4 + \beta_2(1/\exp(dbh/h)) + \beta_3 x^{0{,}1} + \beta_4(1/dbh) + \beta_5 h^Q + \beta_6 x\right]} + \varepsilon_i \quad (4)$$

Where: d_i = diameter (cm) at the height h_i (m) over the stem; dbh = diameter at breast height at 1.3 from the ground; h = total height; $a_{i's}$ and $\beta_{i's}$ = fixed parameters; ε_i = random error; $z = h_i/h; p = 1{,}3/h;\ Q = 1 - z^{1/3};\ x_{1988} = (1-\sqrt{z}/(1-\sqrt{p});$ $x_{2004} = Q/(1-p^{1/3})$.

2.3. Fitting and validation

Fitting was done in three steps. The first was conducting 8 fittings considering four models in the two databases (near thinning and near clear-cut). The non-significant coefficients

($\alpha > 0{,}05$) were removed from models (3) and (4). Two models were selected, one for each database.

The next step was to test the insertion of random terms at the tree level, as described by Liu et al. (2020a). The insertion of the random term aims to generate flexibility for the model (capturing specific characteristics of each tree) and to treat longitudinal data, dealing with the correlation between measurements made along the tree stem in the scaling.

In the third step the best fixed and mixed equations were assessed using the non-parametric Bootstrap validation method with reposition (Efron, 1982) considering 100 random samples. This technique is used to validate equations without stratifying database for fitting and validation, being especially useful for use in small samples, since there is no sample size restriction to apply the methodology (Scolforo et al., 2018; Hall et al., 2019; Santos et al., 2021a).

2.4. Assessment

The assumptions of homogeneity of variances and normality for the residue model were evaluated. Furthermore, the models were assessed using root mean square error (RMSE - 5), mean absolute error (MAE - 6) and mean error (T - 7). Dispersion charts (observed x estimated) were observed as well. Akaike information criterion (AIC), Bayesian information criterion (BIC) and log-likelihood ratio (logLik) were used to compare equations with fixed and mixed equations. Overall fitting behaviour was also assessed by applying the fitted models to a random tree selected in each size class, considering a) trees in which the dbh is smaller or equal to the 1º quartile; b) trees in which the dbh is between the 1º and 3º quartile; c) trees in which the dbh is larger or equal to the third quartile.

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{i=1}^{n}(yo-ye)^2}{n}} \quad (5)$$

$$\mathrm{MAE} = \frac{\sum_{i=1}^{n}|yo-ye|}{n} \quad (6)$$

$$\mathrm{T} = \frac{\sum_{i=1}^{n}(yo-ye)}{n} \quad (7)$$

where: i – positions where diameter was measured over the stem (from1 to n), yo – observed values, ye – estimated values. The other variables were already described.

Data processing was made using the s*oftware* R (R Core Team, 2021). The *nlme* package (Pinheiro et al., 2019) were used to fit mixed models and the package *ggplot2* (Wickham, 2016) was used to produce the charts.

3. RESULTS

3.1. Equations

The four models were fit using the two databases, one composed with trees near thinning, and the other composed with trees near clear-cut ages. The assumptions of homogeneity of variances and normality for the residuals of the model were verified and satisfied. Table 2 shows the significant coefficients of the eight equations produced ($\alpha = 5\%$).

Table 2. Fixed coefficients of the four taper models tested to estimate tree taper for thinning and clear-cut ages.

	Thinning				Clearcut			
Coef.	(1)	(2)*	(3)	(4)	(1)	(2)*	(3)	(4)
a_0	-	-	0.7912	1.1667	-	-	1.5700	1.1546
a_1	-	-	1.1453	0.9070	-	-	0.8623	0.9658
a_2	-	-	0.9916	0.0573	-	-	1.0018	-
β_1	9.8772	0.6356	-	-0.1104	14.6850	-13810.0	2.9854	-
β_2	-49.8424	0.4769	-0.2651	-	-76.0380	13810.0	-1.0204	-0.8217
β_3	110.6273	-60.089	1.3224	0.3980	161.2290	1.361	7.1039	0.5837
β_4	-110.6373	111.1	-0.4717	-0.9339	-152.0940	6.032	-3.7932	-
β_5	41.3019	-1320.0	0.0684	0.0177	53.4230	-6.157	0.1170	0.0796
β_6	-	2347.0	-	0.1021	-	-	-	-0.4682
β_7	-	-1077.0	-	-	-	-	-	-

* The coefficients β_1 to β_7 of equation (2) fitted to the thinning database correspond to the powers 0.00001, 3, 50, 55, 85, 90, and 95, respectively, and the coefficients β_1 to β_5 of equation (2) fitted to the clear-cut database correspond to the powers of 0.00005, 0.00001, 1, 60, and 65.

According to the goodness-of-fit statistics and to the real versus estimated charts, the Kozak equations (3 and 4) overperformed the polynomials equations (1 and 2) (Table 3, Figure 2). Equation (3) was the most accurate for trees near thinning and equation (4) was the most accurate for trees near clear-cut ages.

Table 3. Statistics assessing goodness-of-fit of the equations for trees near thinning and clear-cut ages.

	Thinning			Clear-cut		
Equation	RMSE (cm)	MAE (cm)	T (cm)	RMSE (cm)	MAE (cm)	T (cm)
(1)	1.6812	1.2416	-0.2368	2.7943	2.1860	-0.3535
(2)	1.5528	1.1203	-0.1940	2.7657	2.1478	-0.3524
(3)	1.3274	0.9735	0.0016	2.3464	1.7490	0.0100
(4)	1.3495	0.9922	-0.0062	2.3234	1.7485	0.0037

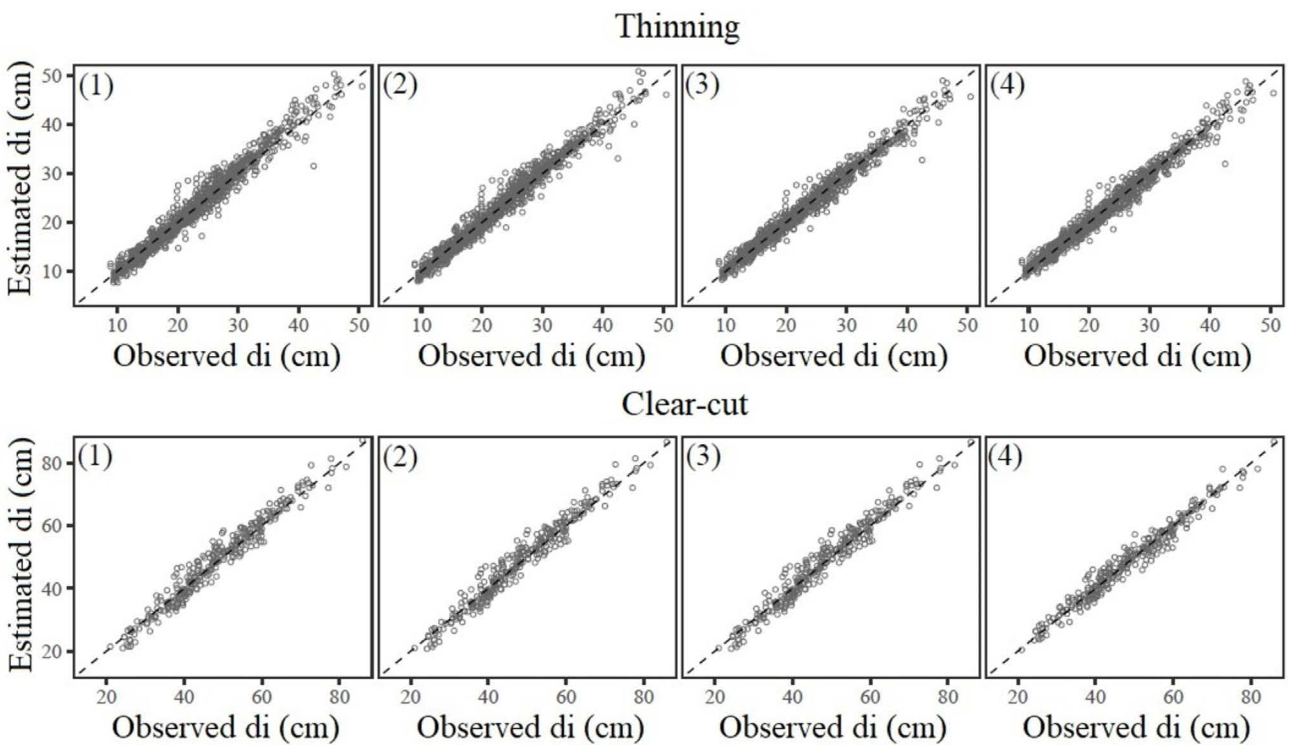

Figure 2. Estimated versus observed diameters over the stem for the four models tested for the thinning and clear-cut databases.

3.2. Fixed and random effects

In the previous step, equations (3) and (4) were selected for trees near thinning and clear-cut, respectively. Next, the insertion of different random terms at tree level were tested. The equations (8) and (9) were selected for trees near thinning and clear cut, respectively. The variance of the random parameters and residues are presented in Table 4.

$$d_i = (0{,}4996 + a_{0i})D^{1{,}3463}0{,}9838^{D}x_{1988}{}^{[-0{,}2660\ \ln(z+0{,}001)+1{,}2856\sqrt{z}-0{,}4523\ \exp(z)+0{,}0609(D/H)]} \quad (8)$$

$$d_i = (0{,}9875 + a_{0i})D^{1{,}0052}x_{2004}{}^{[-1{,}0061(1/\exp\,(D/H))+0{,}6208\ x^{0{,}1}+0{,}0799\ H^{Q}-0{,}4850\ x]} \quad (9)$$

Where: a_{0i} = random term at tree level. The other variables were described previously.

Table 4. Variance of the random parameters and residues of the mixed equations tested

Eq. (8) - Thinning		Eq. (9) - Clear-cut	
a_{0i}	$8.1562*10^{-5}$	a_{0i}	$5.9166*10^{-4}$
ε_i	1.4489	ε_i	4.1185

The mixed approach (Equations 8 and 9) overperformed the fixed approach (Equations 3 e 4) according to the statistics RMSE, MAE, AIC, BIC, and logLik (Table 5). Even though T increased after inserting a random term, the value is still near zero.

Table 5. Root mean square error (RMSE), mean absolute error (MAE), mean error (T), Akaike Information criterion (AIC), Bayesian Information Criterion (BIC) and log-likelihood ratio (logLik) for trees near thinning and clear cut for the mixed and fixed equations.

Thinning						
Equation	**RMSE (cm)**	**MAE (cm)**	**T (cm)**	**AIC**	**BIC**	**logLik**
Fixed equation	1.3274	0.9735	0.0016	3430.531	3469.817	-1707.265
Mixed equation	1.2331	0.9052	0.4035	3313.933	3358.129	-1647.966
Clear-cut						
Equation	**RMSE (cm)**	**MAE (cm)**	**T (cm)**	**AIC**	**BIC**	**logLik**
Fixed equation	2.3234	1.7485	0.0037	1348.563	1374.371	-667.2813
Mixed equation	1.9242	1.4877	0.0336	1322.569	1352.064	-653.2843

Accuracy and behaviour of diameter estimates over the stem using fixed and mixed equations were assessed on a random tree in each size class (Figure 3), showing that the estimates from the mixed equations were more accurate compared to the fixed equations. This is due to the flexibility introduced into the model through the random term that captures variations among trees.

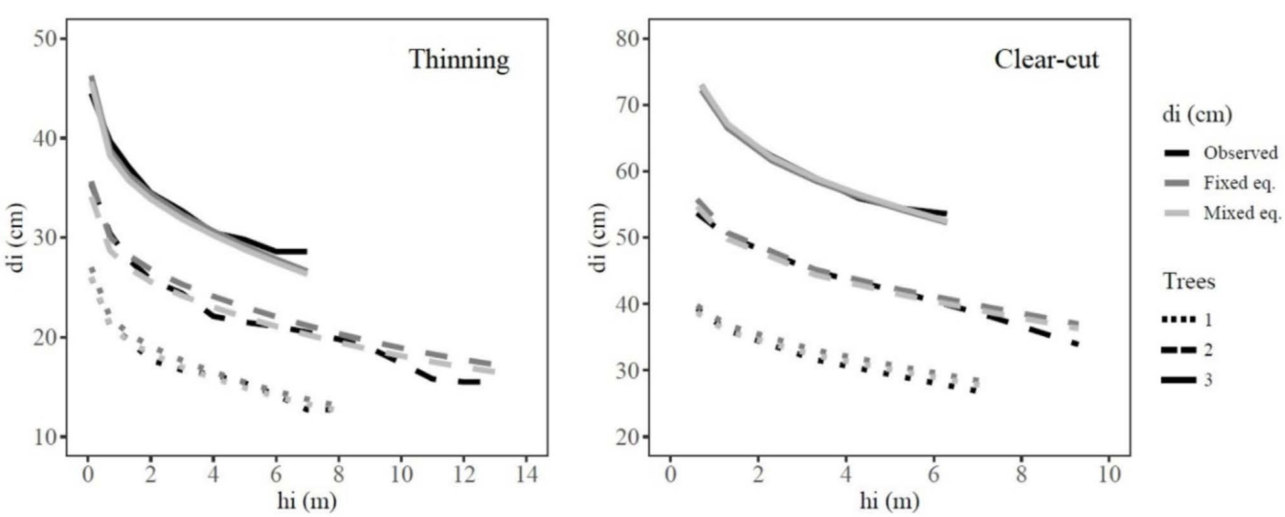

Figure 3. Observed and estimated diameters up to the stem height of *Khaya grandifoliola* for trees from three size classes.

3.3. Validating the models

The selected equations with fixed (3 and 4) and mixed effects (8 and 9) were validated applying the non-parametric Bootstrap method with reposition of 100 samples. The MAE and RMSE distributions are represented on Figure 4. Validation showed high precision and low error, since MAE e RMSE presented short intervals near zero (Figure 4). The distributions associated to the statistics MAE and RMSE of the equations with mixed effects were concentrated more to the left than the fixed effects only equations, indicating greater accuracy compared to the fixed effect equations.

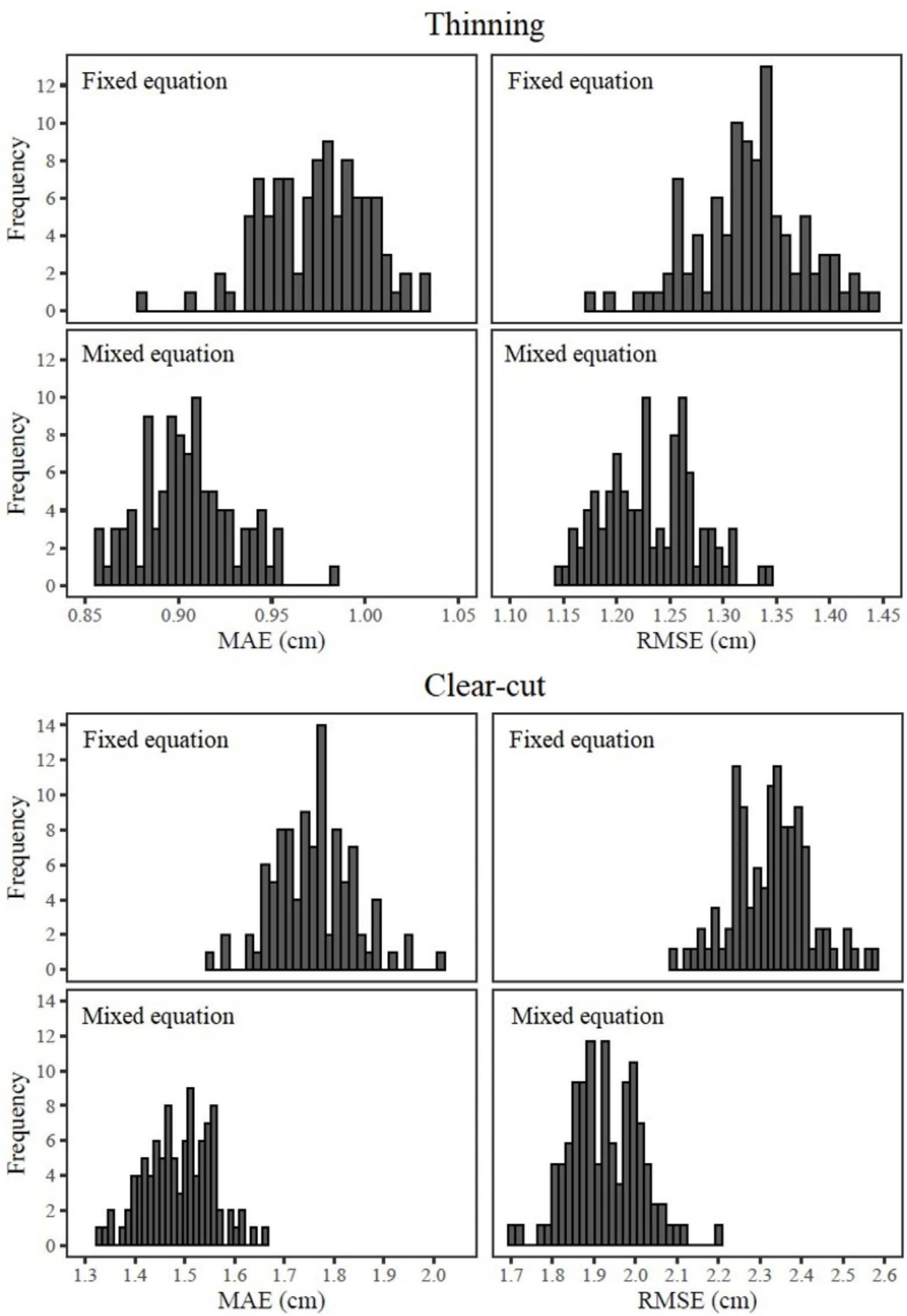

Figure 4. Distribution of MAE and RMSE to 100 random samples tested using the Bootstrap method.

4. DISCUSSION

There are two main reasons that highlight the need to describe tapper for African mahogany trees. First, it produces high-end products, therefore, there is the need to forecast assortments. Second, African mahogany plantation area is growing (ITTO, 2022; Reis et al., 2019; Ribeiro; Ferraz Filho; Scolforo, 2017), which leads to the need of knowing more about this species in terms of increment, shape, and the effect of silvicultural treatments. To model taper precisely, it is important to use a database representing all diameter classes within the forest (Figure 1) and to obtain accurate data over the stem. The indirect scaling applied in this study is a viable solution when it is not possible to harvest trees. Besides, the use of Criterion RD 1000 is validated in the literature (Curto et al., 2019; Stenman et al., 2023). For example, Oliveira et al. (2018) validated the indirect scaling for *Khaya grandifoliola* trees in Minas Gerais used in this study.

To model tree taper allows one to retrieve important aspects related to tree shape, for example the effect of spacing (Vendruscolo et al., 2015), age (Kohler et al., 2016), and genotypes (Souza et al., 2018). Here in this study, it was possible to observe higher decrease in diameters over the stem on younger trees than on older ones, considering three random trees in each size class (Figure 3). This behaviour is in conformity with the pattern described by Kohler et al. (2016), who stated that the stems *Pinus taeda* become more cylindrical with increasing age.

The kind of models considered in this study were the simple models and the variable exponent models. The segmented model from Max and Burkhart (1976) was previously tested in this study and did not perform well because of the nature of the data. Data from indirect scaling was collected up to commercial height, therefore it was not feasible to segment the stem in two or three pieces. The models of the simple kind (Fifth degree polynomial and the integer and fractional powers) were also previously tested using the fitting considering the intercept as in Teo and Esteves (2022) and Kohler et al. (2016), and the fitting with constrains with no intercept as in Scolforo et al. (2018). The models with variable exponent overperformed the simple models (Table 4). Although they are not linear, the variable exponent models are flexible to add covariables in the powers and random variables (Liu et al., 2020a; Liu et al., 2020b). The simple models were not selected but can present high accuracy as in Scolforo et al. (2018).

The inclusion of random effects on variable exponent models increased its accuracy (Table 5). This approach allows models to keep fixed coefficients (mean values associated to the whole database) and random coefficients (values for specific groups). This approach has been used in forest sciences to model tree height (Xie et al., 2020) and volume (Monteiro et al., 2021). The grouping criteria can vary. For example, Santos et al. (2021a) tested inserting random terms in taper models considering three groups (spacings, diameter classes within spacing and trees) and verified more accurate results at the most specific level (trees). In some situations, the fitting approach can allow a generalized fit, with no need to fit models to specific situations (Vendruscolo et al., 2015). Scolforo et al. (2018) developed mixed equations to predict taper for eucalyptus of four genetic families in different areas across Brazil. Santos et al. (2021a) developed a generalized a mixed equation for pine trees planted in two spacings. In this study, data was not continuous in age (7-year-old x 14–15-year-old) so it was not possible to perform a unique fitting, thus two equations were developed.

Beyond fitting, the equations must show accuracy on the validation phase (Figure 4). One alternative to avoid splitting the database into fitting and validation is to apply the Bootstrapping technique, which generates different samples composed by the database (Liu et al. 2020b). The more samples generated, the more the error is close to the normal distribution (Xu and Goodacre, 2018). In this study 100 samples were used to validate the models, as in Hall et al. (2019) and Santos et al. (2021a) modeling growth and taper.

The fixed (3 and 4) and mixed equations (8 and 9) were accurate in the fitting and validation phases. The mixed equations overperformed the fixed one. In this study, we recommend the mixed fitting when scaling data is available and the fixed approach when only inventory data is available (dbh and h). Future studies must be carried out to assess the behaviour of African mahogany tree taper for stands planted with different species, spacings, ages and different areas in the country.

5. CONCLUSION

As we hypothesized, variable exponential models were superior to simple exponential models, and the inclusion of random term was efficient to better the model. Thus, we recommend the variable exponent models with the mixed affect approach with the random term at tree level to model taper of African mahogany trees. For trees with age near thinning, we recommend the mixed equation of Kozak, and for trees near clear-cut, we recommend the Kozak modified equation. These modeling approaches and equations generated here can be used in other *Khaya grandifoliola* plantations with varying ages and areas.

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AUTHORS' CONTRIBUTIONS

Ximena Mendes de Oliveira: Conceptualization (Equal), Data curation (Equal), Formal analysis (Equal), Investigation (Equal), Methodology (Equal), Resources (Equal), Software (Equal), Validation (Equal), Writing - original draft (Equal), Writing - review & editing-(Equal).

Andressa Ribeiro: Conceptualization (Equal), Data curation (Equal), Formal analysis (Equal), Project administration (Equal), Resources (Equal), Supervision (Equal), Visualization (Equal), Writing - review & editing (Equal).

Rafaella Carvalho Mayrinck: Conceptualization (Equal), Data curation (Equal), Methodology (Equal), Resources (Equal), Writing - original draft (Equal), Writing - review & editing (Equal)

Antonio Carlos Ferraz Filho: Conceptualization (Equal), Data curation (Equal), Formal analysis (Equal), Investigation (Equal), Methodology (Equal), Project administration (Equal), Resources (Equal), Supervision (Equal), Validation (Equal), Visualization-Equal, Writing - review & editing (Equal).

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